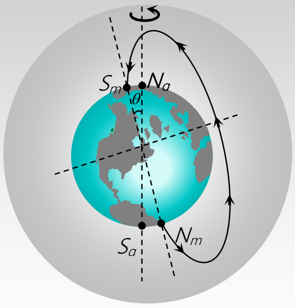
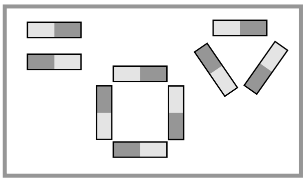
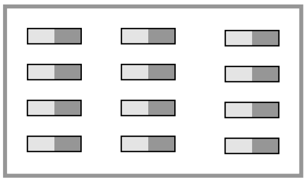


Magnetism

The molecular theory of magnetism was given by Weber and modified later by Ewing. According to this theory.

Every molecule of a substance is a complete magnet in itself. However, in an **magnetised** substance the molecular magnets are randomly oriented to give zero net magnetic moment. On magnetising, the molecular magnets are realigned in a specific direction leading to a net magnetic moment.

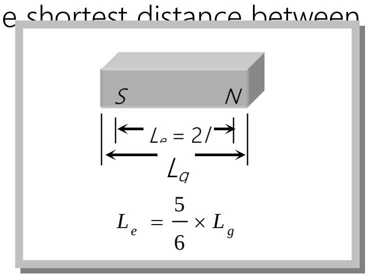

(Unmagnetised)

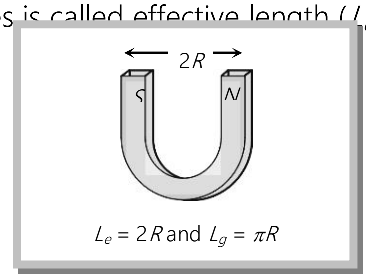

(Magnetised)

Note : ❑ On heating/hammering the magnetism of magnetic substance reduces.

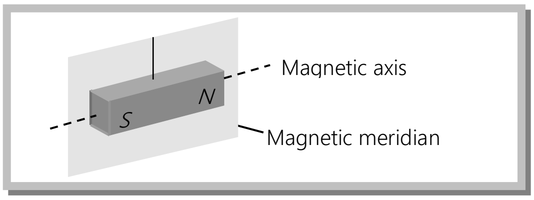
Bar Magnet

A bar magnet consist of two equal and opposite magnetic pole separated by a small distance. Poles are not exactly at the ends. The shortest distance between two poles is called effective length (L_e) and is less then its geometric length (L_g).

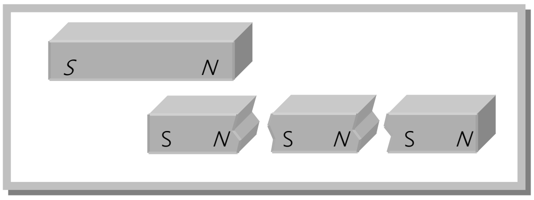




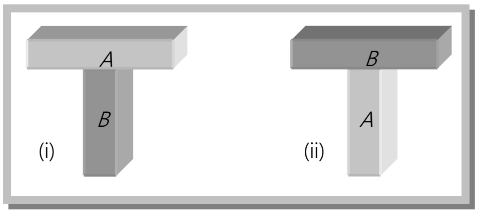
(1) **Directive properties** : When a magnet suspended freely it stays in the earth's *N*-*S* direction (in magnetic meridian).



(2) **Monopole concept** : If a magnet is Broken into number of pieces, each piece becomes a magnet. This in turn implies that monopoles do not exist. (*i.e.*, ultimate individual unit of magnetism in any magnet is called dipole).



(3) For two rods as shown, if both the rods attract in case (i) and doesn't attract in case (ii) then, *B* is a magnetic and *A* is simple iron rod. Repulsion is sure test of magnetism.



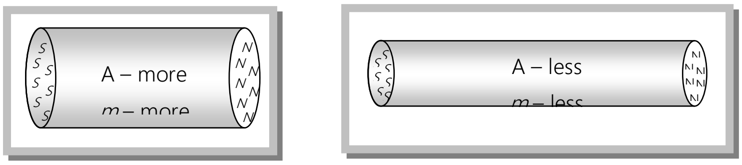
(4) **Pole strength (*m*)** : The strength of a magnetic pole to attract magnetic materials towards itself is known as pole strength.

(i) It is a scalar quantity.

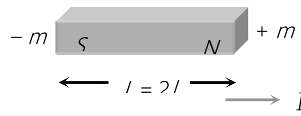
(ii) Pole strength of *N* and *S* pole of a magnet is conventionally represented by $+m$ and $-m$ respectively.

(iii) It's SI unit is $amp \times m$ or *N*/ *Tesla* and dimensions are $[LA]$.

(iv) Pole strength of the magnet depends on the nature of material of magnet and area of cross section. It doesn't depends upon length.



(5) **Magnetic moment or magnetic dipole moment $(\vec{M})$** : It represents the strength of magnet. Mathematically it is defined as the product of the strength of either pole and effective length. *i.e.* $\vec{M} = m(2\vec{l})$

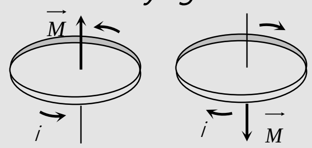


(i) It is a vector quantity directed from south to north.

(ii) It's S.I. unit $amp \times m^2$ or *N-m/ Tesla* and dimensions $[AL^2]$

(iii) Magnetic moment in various other situations.

Current carrying coil

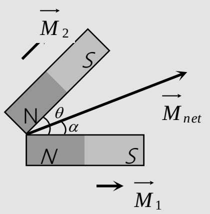


Magnetic moment $M = NiA$

N = number of turns,

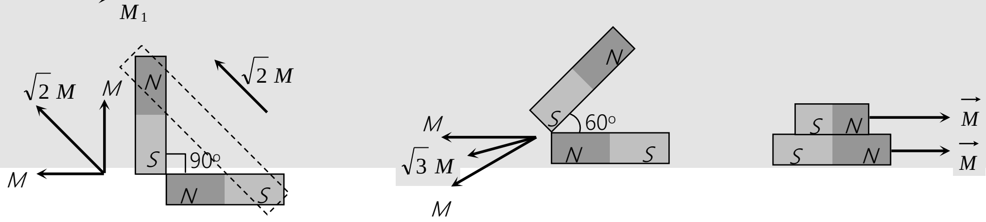
i = current through the coil, A = Area of the coil

Combination of bar magnet



$$M_{net} = \sqrt{M_1^2 + M_2^2 + 2M_1M_2\cos\theta}$$

$$\tan\alpha = \frac{M_2 \sin\theta}{M_1 + M_2\cos\theta}$$



$M_{net} = 2M$

Revolving charge

(a) Orbital electron : In an atom electrons revolve around the nucleus in circular orbit and it is equivalent to the flow of current in the orbit. Thus the orbit of electrons is considered as tiny current loop with magnetic moment.

$M = e\nu A = \frac{e\omega r^2}{2} = \frac{1}{2}evr = \frac{e}{2m}L = \frac{eh}{4\pi m}$; where, ω = angular speed, ν = frequency, v = linear speed and L = Angular moment $I\omega$.

(b) For geometrical symmetrical charged rotating bodies : The magnetic moment given by $M = \frac{QL}{2m} = \frac{QI\omega}{2m}$; where m = mass of rotating body, Q = charge on body, I = moment of inertia of rotating body about axis of rotation.

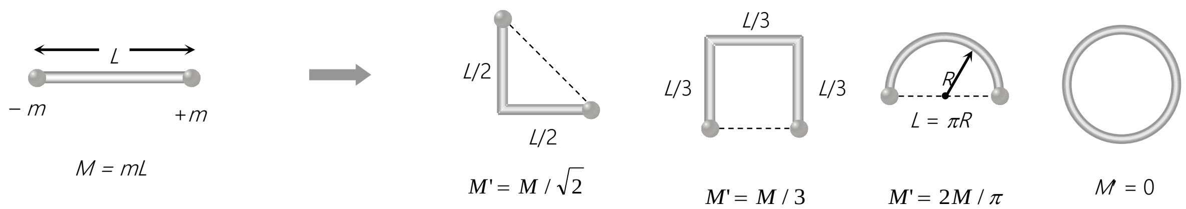
$I = \frac{mR^2}{2}$, $M = \frac{1}{4}Q\omega R^2$ $I = MR^2$, $M = \frac{1}{2}Q\omega R^2$ $I = \frac{mL^2}{12}$, $M = \frac{1}{24}Q\omega L^2$

Note : ❑ Bohr magneton $\mu_B = \frac{eh}{4\pi m} = 9.27\times10^{-24}\ A/m^2$. It serves as natural unit of magnetic moment. Bohr magneton can be defined as the orbital magnetic moment of an electron circulating in inner most orbit.

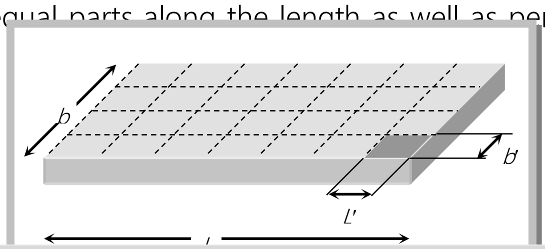
❑ Magnetic moment of straight current carrying wire is zero.

❑ Magnetic moment of toroid is zero.

❑ If a magnetic wire of magnetic moment (M) is bent into any shape then it's M decreases as it's length (L) always decreases and pole strength remains constant.



(6) **Cutting of a bar magnet :** Suppose we have a rectangular bar magnet having length, breadth and mass are L, b and w respectively if it is cut in n equal parts along the length as well as perpendicular to the length simultaneously as shown in the figure then

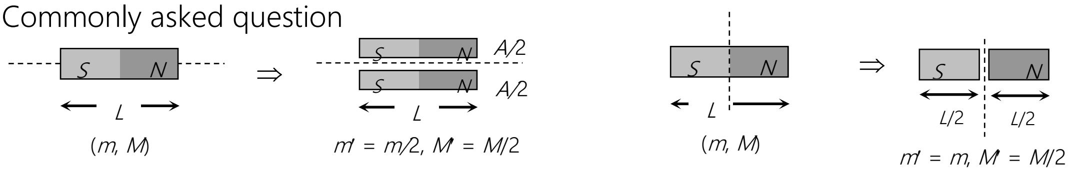


Length of each part $L' = \frac{L}{\sqrt{n}}$, breadth of each part $b' = \frac{b}{\sqrt{n}}$, Mass of each part $w' = \frac{w}{n}$, pole strength of each part $m' = \frac{m}{\sqrt{n}}$, Magnetic moment of each part $M' = m'L' = \frac{m}{\sqrt{n}} \times \frac{L}{\sqrt{n}} = \frac{M}{n}$

If initially moment of inertia of bar magnet about the axes passing from centre and perpendicular to it's length is $I = w\left(\frac{L^2 + b^2}{12}\right)$ then moment of inertia of each part $I' = \frac{I}{n^2}$

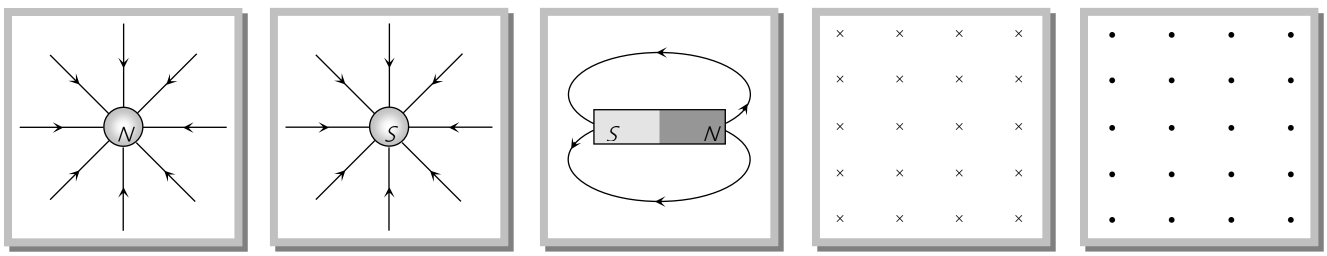
Note : ❑ For short bar magnet $b = 0$ so $L' = \frac{L}{n}, w' = \frac{w}{n}, m' = m, M' = \frac{M}{n}$ and $I' = \frac{I}{n^3}$

❑ Commonly asked question



Various Terms Related to Magnetism

(1) **Magnetic field and magnetic lines of force :** Space around a magnetic pole or magnet or current carrying wire within which it's effect can be experienced is defined magnetic field. Magnetic field can be represented with the help of a set of lines or curves called magnetic lines of force.



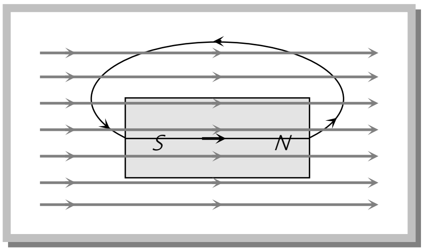
Isolated north pole | Isolated south pole | Magnetic dipole | Inward magnetic field | Outward magnetic field

(2) **Magnetic flux (ϕ) and flux density (B)**

(i) The number of magnetic lines of force passing normally through a surface is defined as magnetic flux (ϕ). It's S.I. unit is *weber* (*wb*) and CGS unit is *Maxwell*.

Remeber 1 *wb* = 10^8 *maxwell*.

(ii) When a piece of a magnetic substance is placed in an external magnetic field the substance becomes magnetised. The number of magnetic lines of induction inside a magnetised substance crossing unit area normal to their direction is called magnetic induction or magnetic flux density $(\vec{B})$. It is a vector quantity.



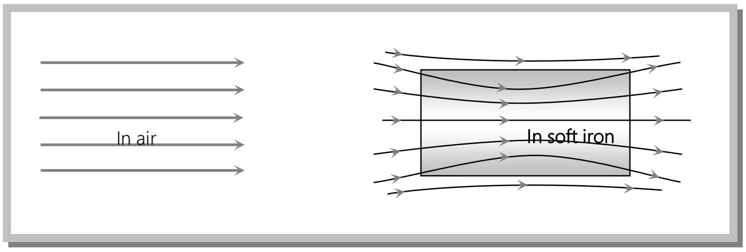
It's SI unit is *Tesla* which is equal to $\frac{wb}{m^2} = \frac{N}{amp \times m} = \frac{J}{amp \times m^2} = \frac{volt \times \sec}{m^2}$

and CGS unit is *Gauss*. Remember 1 *Tesla* = 10^4 *Gauss*.

Note : ❑ Magnetic flux density can also be defined in terms of force experienced by a unit north pole placed in that field *i.e.* $B = \frac{F}{m_0}$.

(3) **Magnetic permeability :** It is the degree or extent to which magnetic lines of force can enter a substance and is denoted by μ. or

Characteristic of a medium which allows magnetic flux to pass through it is called it's permeability. *e.g.* permeability of soft iron is 1000 times greater than that of air.



Also $\mu = \mu_0\,\mu_r$; where μ_0 = absolute permeability of air or free space = $4\pi \times 10^{-7}\, tesla \times m / amp$.

and μ_r = Relative permeability of the medium = $\frac{B}{B_0} = \frac{\text{flux density in material}}{\text{flux density in vacuum}}$.

(4) **Intensity of magnetising field $(\vec{H})$ (magnetising field)** : It is the degree or extent to which a magnetic field can magnetise a substance. Also $H = \frac{B}{\mu}$.

It's SI unit is $A/m. = \frac{N}{m^2 \times Tesla} = \frac{N}{wb} = \frac{J}{m^3 \times Tesla} = \frac{J}{m \times wb}$ It's CGS unit is *Oersted*. Also 1*oersted* = 80 *A/m*

(5) **Intensity of magnetisation (*I*)** : It is the degree to which a substance is magnetised when placed in a magnetic field.

It can also be defined as the pole strength per unit cross sectional area of the substance or the induced dipole moment per unit volume.

Hence $I = \frac{m}{A} = \frac{M}{V}$. It is a vector quantity, it's S.I. unit is *Amp/m*.

(6) **Magnetic susceptibility (χ_m)** : It is the property of the substance which shows how easily a substance can be magnetised. It can also be defined as the ratio of intensity of magnetisation (*I*) in a substance to the magnetic intensity (*H*) applied to the substance, *i.e.* $\chi_m = \frac{I}{H}$. It is a scalar quantity with no units and dimensions.

(7) **Relation between permeability and susceptibility** : Total magnetic flux density B in a material is the sum of magnetic flux density in vacuum B_0 produced by magnetising force and magnetic flux density due to magnetisation of material B_m. *i.e.* $B = B_0 + B_m$

$\Rightarrow B = \mu_0 H + \mu_0 I = \mu_0 (H + I) = \mu_0 H(1 + \chi_m)$. Also $\mu_r = (1 + \chi_m)$

Note : ❑ In CGS $B = H + 4\pi I$ and $\mu = 1 + 4\pi\chi_m$.

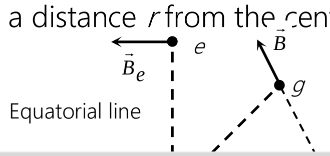
Force and Field

(1) **Coulombs law in magnetism** : The force between two magnetic poles of strength m_1 and m_2 lying at a distance r is given by $F = k.\frac{m_1 m_2}{r^2}$. In S.I. units $k = \frac{\mu_0}{4\pi} = 10^{-7} wb / Amp \times m$, In CGS units $k = 1$

(2) **Magnetic field**

(i) Magnetic field due to an imaginary magnetic pole (Pole strength *m*) : Is given by $B = \frac{F}{m_0}$ also $B = \frac{\mu_0}{4\pi} . \frac{m}{d^2}$

(ii) Magnetic field due to a bar magnet : At a distance *r* from the centre of magnet



(a) On axial position

$$B_a = \frac{\mu_0}{4\pi}\frac{2Mr}{(r^2 - l^2)^2};$$

If $l << r$ then $B_a = \frac{\mu_0}{4\pi}\frac{2M}{r^3}$

(b) On equatorial position : $B_e = \frac{\mu_0}{4\pi}\frac{M}{(r^2 + l^2)^{3/2}}$; If $l << r$; then $B_e = \frac{\mu_0}{4\pi}\frac{M}{r^3}$

(c) General position : In general position for a short bar magnet $B_g = \frac{\mu_0}{4\pi}\frac{M}{r^3}\sqrt{(3\cos^2\theta + 1)}$

(3) **Bar magnet in magnetic field** : When a bar magnet is left free in an uniform magnetic field, if align it self in the directional field.

(i) Torque : $\tau = MB\sin\theta \Rightarrow \vec{\tau} = \vec{M} \times \vec{B}$

(ii) Work : $W = MB(1 - \cos\theta)$

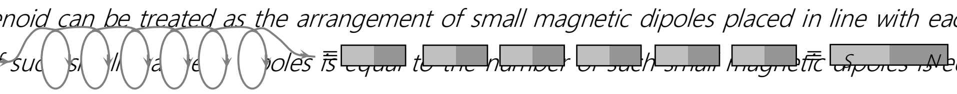
(iii) Potential energy : $U = MB\cos\theta = -\vec{M}.\vec{B}$; ($\theta$ = Angle made by the dipole with the field)

Note : ❑ For more details see comparative study of electric and magnetic dipole in electrostatics.

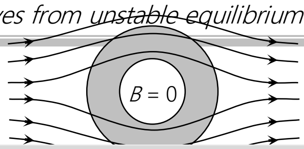
(4) **Gauss's law in magnetism** : Net magnetic flux through any surface is always zero *i.e.* $\oint \vec{B}.\vec{ds} = 0$

Concepts

- *The property of magnetism is materials is on account of magnetic moment in the material.*
- *Atoms which have paired electron have the magnetic moment zero.*
- *Magnetostriction : The length of an iron bar changes when it is magnetised, when an iron bar magnetised it's length increases due to alignment of spins parallel to the field. This increase is in the direction of magnetisation. This effect is known as magnetostriction.*
- *A current carrying solenoid can be treated as the arrangement of small magnetic dipoles placed in line with each other as shown. The number of such small magnetic dipoles is equal to the number of such small magnetic dipoles is equal to the number of turns in the in the solvent*



- *When a magnetic dipole of moment M moves from unstable equilibrium to stable equilibrium position in a magnetic field B,*



the kinetic energy by it will be 2 MB.

☞ *Intensity of magnetisation (I) is produced in materials due to spin motion of electrons.*

☞ *For protecting a sensitive equipment from the external magnetic field it should be placed inside an iron cane. (magnetic shielding)*

Examples

Example. 1 The work done in turning a magnet of magnetic moment M by an angle of 90° form the meridian, is n times the corresponding work done to turn it through an angle of 60°. The value of *n* is given by[MP PET 2003]

(a) 2 (b) 1 (c) 1/2 (d) 1/4

Solution : (a) $W = MB(1-\cos\theta) \Rightarrow W_{0^o \to 90^o} = nx\left(W_{0^o \to 60^o}\right) \Rightarrow MB(1-\cos 90^o) = n \times MB(1-\cos 60^o) \Rightarrow n = 2$

Example. 2 The magnetic susceptibility of a material of a rod is 499, permeability of vacuum is $4\pi \times 10^{-7} H/m$. Permeability of the material of the rod in *henry/metre* is

(a) $\pi \times 10^{-4}$ (b) $2\pi \times 10^{-4}$ (c) $3\pi \times 10^{-4}$ (d) $4\pi \times 10^{-4}$

Solution : (b) $\mu_r = (1+\chi_m) \Rightarrow \mu_r = (1+499) = 500$ Also $\mu = \mu_0 \mu_r = 4\pi \times 10^{-7} \times 500 = 2\pi \times 10^{-4}$

Example. 3 A magnetic needle lying parallel to a magnetic field requires *W* units of work to turn it through 60°. The torque required to maintain the needle in this position will be

(a) $\sqrt{3}\, W$ (b) $-W$ (c) $\frac{\sqrt{3}}{2} W$ (d) $2W$

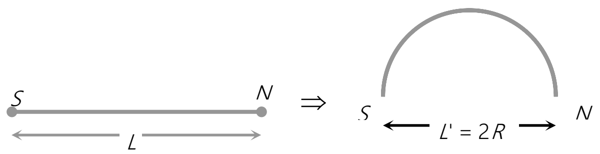
Solution : (a) $\tau = MB\sin\theta$ and $W = MB(1-\cos\theta) \Rightarrow W = MB(1-\cos 60^o) = \frac{MB}{2}$. Hence $\tau = MB\sin 60^o = \frac{\sqrt{3}MB}{2} = \sqrt{3}W$

Example. 4 An iron rod of length *L* and magnetic moment *M* is bent in the form of a semicircle. Now its magnetic moment will be [CPMT 1984; MP Board 1986; NCERT 1975; MP PET/PMT 1988; EAMCET (Med.) 1995; Manipal MEE 1995; RPMT 1996; BHU 1995; MP PMT 2002]

(a) M (b) $\frac{2M}{\pi}$ (c) $\frac{M}{\pi}$ (d) $M\pi$

Solution : (b) On bending a rod it's pole strength remains unchanged where as it's magnetic moment changes

New magnetic moment $M' = m(2R) = m\left(\frac{2L}{\pi}\right) = \frac{2M}{\pi}$



Example: 5 A short bar magnet with its north pole facing north forms a neutral point at *P* in the horizontal plane. It the magnet is rotated by 90° in the horizontal plane, the net magnetic induction at *P* is : (Horizontal component of earth's magnetic field = B_H)

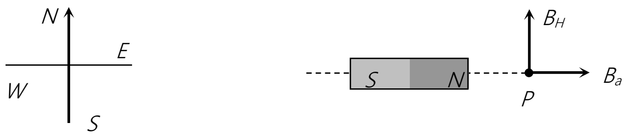
(a) 0 (b) $2\,B_H$ (c) $\frac{\sqrt{5}}{2}B_H$ (d) $\sqrt{5}\,B_H$

Solution : (d) Initially

Neutral point is obtained on equatorial line and at neutral point $|B_H| = |B_e|$

Where B_H = Horizontal component of earth's magnetic field and Be = Magnetic field due to bar magnet on it's equatorial line

Finally



Point *P* comes on axial line of the magnet and at *P*, net magnetic field

$$B = \sqrt{B_H^2 + B_a^2} = \sqrt{(2Be)^2 + (B_H)^2} = \sqrt{(2B_H)^2 + B_H^2} = \sqrt{5}B_H$$

Example: 6 A bar magnet of magnetic moment 3.0 *Amp* × *m* is placed in a uniform magnetic induction field of 2×10^{-5} *T*. If each pole of the magnet experiences a force of $6\times10^{-4}N$ the length of the magnet is

(a) 0. 5 *m* (b) 0. 3 *m* (c) 0.2 *m* (d) 0.1 *m*

Solution : (d) $M = mL$ and $F = mB$ $,\Rightarrow F = \frac{M}{L}\times B \Rightarrow 6\times10^{-4} = \frac{3}{L}\times2\times10^{-5} \Rightarrow L = 0.1m$

Example: 7 Force between two identical bar magnets whose centres are *r metre* apart is 4.8 *N* when their axes are in the same line. If the separation is increases to 2*r metre*, the force between them is reduced to

[AIIMS 1995; Pb. CET 1997]

(a) 2.4 *N* (b) 1.2 *N* (c) 0.6 *N* (d) 0.3 *N*

Solution : (d) Force between two bar magnet $F \propto \frac{1}{d^4} \Rightarrow \frac{F_1}{F_2} = \left(\frac{d_2}{d_1}\right)^4 \Rightarrow \frac{4.8}{F_2} = \left(\frac{2r}{r}\right)^2 \Rightarrow F_2 = 0.3N$ Where *d*= separation between magnets.

Example: 8 Two identical magnetic dipoles of magnetic moments 1.0 *A-m*² each, placed at a separation of 2m with their axis perpendicular to each other. The resultant magnetic field at a point midway between the dipoles is

[Roorkee 1995]

(a) $5\times10^{-7}T$ (b) $\sqrt{5}\times10^{-7}T$ (c) $\frac{T}{2}$ (d) None of these

Solution : (b) $B_1=\frac{\mu_0}{4\pi}.\frac{2M}{\left(\frac{r}{2}\right)^3}=10^{-7}\times\frac{2\times1}{\left(\frac{2}{2}\right)^3}$; $B_2=\frac{\mu_0}{4\pi}\frac{M}{\left(\frac{r}{2}\right)^3}=10^{-7}\times\frac{1}{\left(\frac{2}{2}\right)^3}$; $B_{net}=\sqrt{(2\times10^{-7})^2+(10^{-7})^2}=\sqrt{5}\times10^{-7}T$

Example. 9 A magnet of magnetic moment 20 C.G.S. units is freely suspended in a uniform magnetic field of intensity 0.3 C.G.S. units. The amount of work done in deflecting it by an angle of 30° in C.G.S. units is [MP PET 1991]

(a) 6 (b) $3\sqrt{3}$ (c) $3(2-\sqrt{3})$ (d) 3

Solution : (c) $W=MB(1-\cos\theta)\Rightarrow W=20\times0.3(1-\cos30^o)=3(2-\sqrt{3})$

Example. 10 The magnetic field at a point *X* on the axis of a small bar magnet is equal to the field at a point *Y* on the equator of the same magnet. The ratio of the distance of *X* and *Y* from the centre of the magnet is

[MP PMT 1990]

(a) 2^{-3} (b) $2^{-1/3}$ (c) 2^3 (d) $2^{1/3}$

Solution : (d) Suppose distances of points *X* and *Y* from magnet are *x* and *y* respectively then According to question

$B_{axial}=B_{\text{equatorial}}\Rightarrow\frac{\mu_0}{4\pi}.\frac{2M}{x^3}=\frac{\mu_0}{4\pi}.\frac{M}{y^3}\Rightarrow\frac{x}{y}=\frac{2^{1/3}}{1}$

Example. 11 A magnetising field of 2000 *A/m* produces a flux 6.28×10^{-4} *weber* in a rod. If the area of cross-section is $2\times10^{-5}m^2$. Then the relative permeability of the substance is

(a) 0.75×10^{-2} (b) 1.25×10^4 (c) 0.25 (d) 1.01

Solution : (b) By using $B=\mu_0\mu_rH$ and $B=\frac{\phi}{A}$, $\Rightarrow\mu_r=\frac{\phi}{A\mu_0H}=\frac{6.28\times10^{-4}}{2\times10^{-5}\times4\pi\times10^{-7}\times2000}=1.25\times10^4$

Example. 12 Due to a small magnet intensity at a distance *x* in the end on position is 9 Gauss. What will be the intensity at a distance $\frac{x}{2}$ on broad side on position

(a) 9 *Gauss* (b) 4 *Gauss* (c) 36 *Gauss* (d) 4.5 *Gauss*

Solution : (c) In C.G.S. $B_{axial}=9=\frac{2M}{x^3}$ .....(i) $B_{\text{equaterial}}=\frac{M}{\left(\frac{x}{2}\right)^3}=\frac{8M}{x^3}$ .....(ii)

From equation (i) and (ii) $B_{\text{equaterial}}=36$ *Gauss*.

Example. 13 The magnetic moment produced in a substance of $1gm$ is $6\times10^{-7}ampere-metre^2$. If its density is $5\,gm/cm^3$, then the intensity of magnetisation in *A/m* will be

(a) 8.3×10^6 (b) 3.0 (c) 1.2×10^{-7} (d) 3×10^{-6}

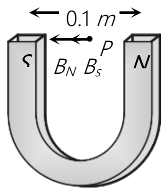
Solution : (b) $I=\frac{M}{V}=\frac{M}{\text{mass/density}}$, given mass = *1gm* = $10^{-3}kg$, and density $=5gm/cm^3=\frac{5\times10^{-3}kg}{(10^{-2})^3m^3}=5\times10^3kg/m^3$

Hence $I=\frac{6\times10^{-7}\times5\times10^3}{10^{-3}}=3$

Example. 14 The distance between the poles of a horse shoe magnet is 0.1 *m* and its pole strength is 0.01 *amp-m*. The induction of magnetic field at a point midway between the poles will be

(a) $2\times10^{-5}T$ (b) $4\times10^{-6}T$ (c) $8\times10^{-7}T$ (d) Zero

Solution : (c) Net magnetic field at mid point *P*, $B = B_N + B_S$



where B_N = magnetic field due to *N*- pole

B_S = magnetic field due to *S*- pole

$$B_N = B_S = \frac{\mu_0}{4\pi}\frac{m}{r^2} = 10^{-7}\times\frac{0.01}{\left(\frac{0.1}{2}\right)^2} = 4\times10^{-7}T \quad \therefore B_{net} = 8\times10^{-7}T.$$

Example. 15 A cylindrical rod magnet has a length of 5 cm and a diameter of 1 cm. It has a uniform magnetisation of 5.30 × 10^3 *Amp*/m^3. What its magnetic dipole moment

(a) $1\times10^{-2}J/T$ (b) $2.08\times10^{-2}J/T$ (c) $3.08\times10^{-2}J/T$ (d) $1.52\times10^{-2}J/T$

Solution : (b) Relation for dipole moment is, $M = I\times V$, Volume of the cylinder $V = \pi r^2 l$, Where *r* is the radius and *l* is the length of the cylinder, then dipole moment,

$$M = I\times\pi r^2 l = (5.30\times10^3)\times\frac{22}{7}\times(0.5\times10^{-2})^2(5\times10^{-2}) = 2.08\times10^{-2}J/T$$

Example. 16 A bar magnet has a magnetic moment of $2.5JT^{-1}$ and is placed in a magnetic field of $0.2T$. Work done in turning the magnet from parallel to anti-parallel position relative to field direction is

(a) $0.5J$ (b) $1J$ (c) $2J$ (d) $0J$

Solution : (b) Work done, $W = -MB(\cos\theta_2 - \cos\theta_1) = -MB(\cos 180^o - \cos 0^o) = -MB(-1-1) = 2MB = 2\times2.5\times0.2 = 1J$

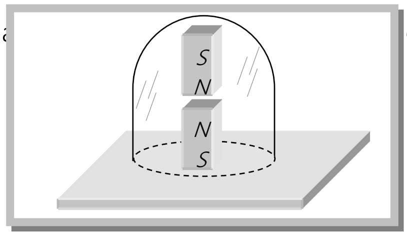
Example. 17 A bar magnet with it's poles 25 *cm* apart and of pole strength 24 *amp*×*m* rests with it's centre on a frictionless pivot. A force *F* is applied on the magnet at a distance of 12 *cm* from the pivot so that it is held in equilibrium at an angle of 30° with respect to a magnetic field of induction 0.25 *T*. The value of force *F* is

(a) 5.62 *N* (b) 2.56 *N* (c) 6.52 *N* (d) 6.25 *N*

Solution : (d) In equilibrium

Magnetic torque = Deflecting torque $\Rightarrow MB \sin\theta = F.d$ or $F = \frac{mlB \sin\theta}{d}$ $\frac{24 \times 0.25 \times 0.25 \sin 30^o}{0.12} = 6.25\,N$

Example: 18 Two identical bar magnets with a length 10 *cm* and weight 50 *gm – weight* are arranged freely with their like poles facing in a arranged vertical glass tube. The upper magnet hangs in the air above the lower one so that the distance between the nearest pole of the ma… e poles of each magnet will be



(a) 6.64 *amp* × *m*

(b) 2 *amp* × *m*

(c) 10.25 *amp* × *m*

(d) None of these

Solution : (a) The weight of upper magnet should be balanced by the repulsion between the two magnet

$$\therefore \frac{\mu}{4\pi}.\frac{m^2}{r^2} = 50\,gm - wt \quad \Rightarrow 10^{-7} \times \frac{m^2}{(9 \times 10^{-6})} = 50 \times 10^{-3} \times 9.8 \Rightarrow m = 6.64 amp \times m$$

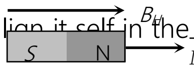
Tricky Example: 1

A bar magnet of magnetic moment 2.0 *A-m*² is free to rotate about a vertical axis passing through its centre. The magnet is released form rest from east–west position. Then the kinetic energy of the magnet as it takes north-south position is (Horizontal component of earth's field is $25\,\mu T$)

[EAMCET (Engg.) 1996]

(a) $25\,\mu J$ (b) $50\,\mu J$ (c) $100\,\mu J$ (d) $12.5\,\mu J$

Solution : (b) When a bar magnet suspended freely in earth's magnetic field, it always align it self in the direction of field (*i.e.* along *N*– *S* direction)

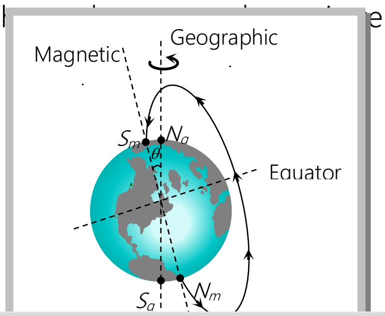


So by using $U = -MB_H \cos\theta$; where θ = angle between M and B_H

$U = -M \times B \cos 0 = -2 \times 25 = -50\,\mu J$

Earth's magnetic Field (Terrestrial Magnetism)

As per the most established theory it is due to the rotation of the earth where by the various charged ions present in the molten state in the core of th… e a current.



(1) The magnetic field of earth is similar to one which would be obtained if a huge magnet is assumed to be buried deep inside the earth at it's centre.

(2) The axis of rotation of earth is called geographic axis and the points where it cuts the surface of earth are called geographical poles (N_g, S_g). The circle on the earth's surface perpendicular to the geographical axis is called equator.

(3) A vertical plane passing through the geographical axis is called geographical meridian.

(4) The axis of the huge magnet assumed to be lying inside the earth is called magnetic axis of the earth. The points where the magnetic axis cuts the surface of earth are called magnetic poles. The circle on the earth's surface perpendicular to the magnetic axis is called magnetic equator.

(5) Magnetic axis and Geographical axis don't coincide but they makes an angle of 17.5° with each other.

(6) Magnetic equator divides the earth into two hemispheres. The hemisphere containing south polarity of earth's magnetism is called northern hemisphere while the other, the southern hemisphere.

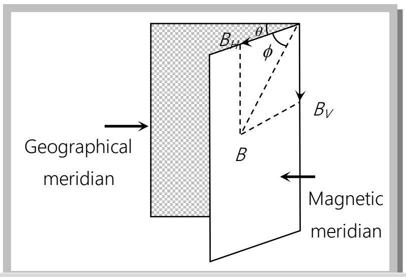
(7) The magnetic field of earth is not constant and changes irregularly from place to place on the surface of the earth and even at a given place in varies with time too.

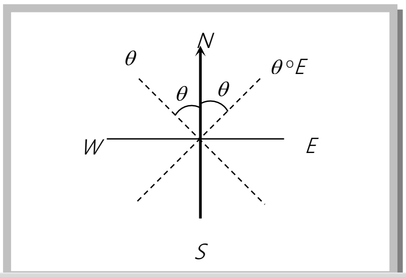
(8) Direction of earth's magnetic field is from S (geographical south) to N (Geographical north).

Elements of Earth's Magnetic Field

The magnitude and direction of the magnetic field of the earth at a place are completely given by certain. quantities known as magnetic elements.

(1) **Magnetic Declination (θ) :** It is the angle between geographic and the magnetic meridian planes.





Declination at a place is expressed at $\theta^o E$ or $\theta^o W$ depending upon whether the north pole of the compass needle lies to the east or to the west of the geographical axis.

(2) **Angle of inclination or Dip (ϕ) :** It is the angle between the direction of intensity of total magnetic field of earth and a horizontal line in the magnetic meridian.

(3) **Horizontal component of earth's magnetic field (B_H) :** Earth's magnetic field is horizontal only at the magnetic equator. At any other place, the total intensity can be resolved into horizontal component (B_H) and vertical component (B_V).

Also $B_H = B\cos\phi$ ...... (i) and $B_V = B\sin\phi$ .... (ii)

By squaring and adding equation (i) and (ii) $B = \sqrt{B_{H^2} + B_{V^2}}$

Dividing equation (ii) by equation (i) $\tan\phi = \frac{B_V}{B_H}$

Note : ❑ At equator $\theta = 0 \Rightarrow B_H = B, B_V = 0$ while at poles $\phi = 90^o \Rightarrow B_H = 0, B_V = B.$

Magnetic Maps and Neutral Points

(1) Magnetic maps (*i.e.* Declination, dip and horizontal component) over the earth vary in magnitude from place to place. It is found that many places have the same value of magnetic elements. The lines are drawn joining all place on the earth having same value of a magnetic elements. These lines forms magnetic map.

(i) Isogonic lines: These are the lines on the magnetic map joining the places of equal declination.

(ii) Agonic line: The line which passes through places having zero declination is called agonic line.

(iii) Isoclinic lines : These are the lines joining the points of equal dip or inclination.

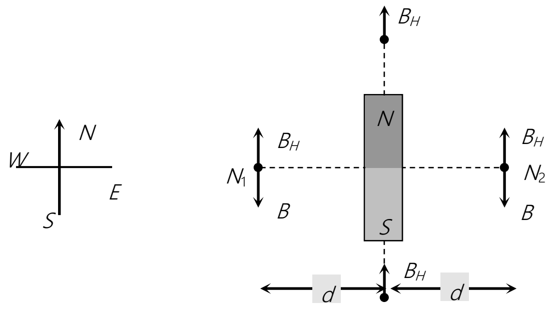
(iv) Aclinic line : The line joining places of zero dip is called aclinic line (or magnetic equator)

(v) Isodynamic lines : The lines joining the points or places having the same value of horizontal component of earth's magnetic field are called isodynamic lines.

(2) **Neutral points :** At the neutral point, magnetic field due to the bar magnet is just equal and opposite to the horizontal component of earth's magnetic field.

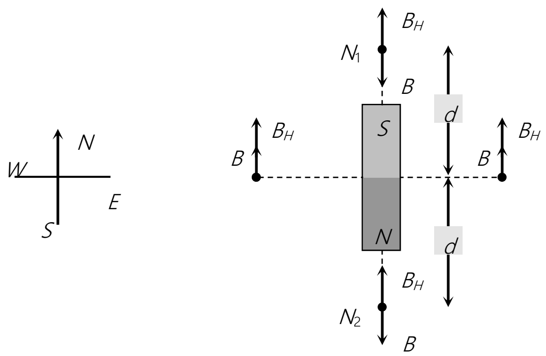
(i) Magnet is placed horizontally in a horizontal plane.

N- pole of magnet is facing N- pole of earth



Two neutral points N_1 and N_2 are obtained on equatorial line of bar magnet as shown and at Neutral points $B = B_H \Rightarrow \frac{\mu_0}{4\pi}\frac{M}{d^3} = B_H$

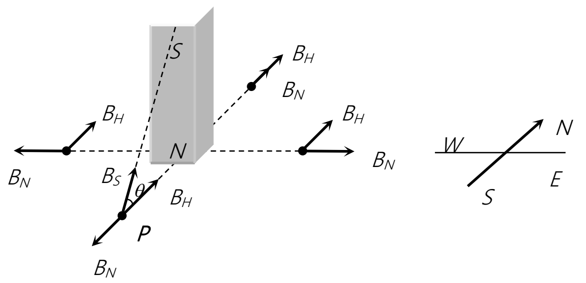
N - pole of magnet is facing N- pole of earth



Two neutral points N_1 and N_2 are obtained on axial line of B or magnet and at neutral points $B = B_H$ i.e. $\frac{\mu_0}{4\pi}.\frac{2M}{d^3} = B_H$

(ii) **Magnet is placed vertically in a horizontal plane**

N- pole of magnet is the horizontal plane



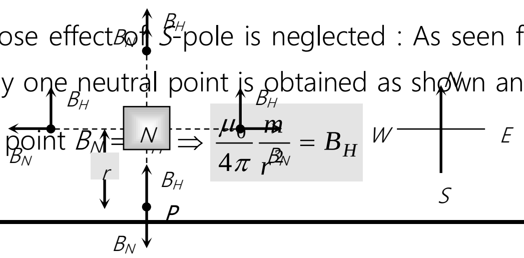
B_N = Magnetic field due to N-pole

B_S = Magnetic field due to S-pole

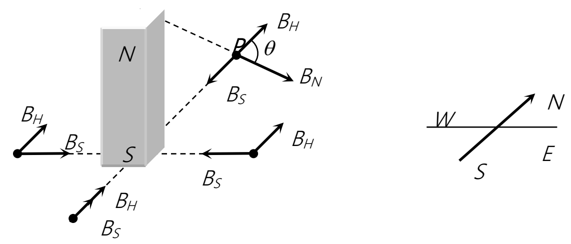
M = Pole strength of each pole of the magnet

At neutral point P: $B_N - B_S \cos\theta = B_H \quad (B_S < B_N)$

If suppose effect of S-pole is neglected : As seen from top only one neutral point is obtained as shown and at neutral point $B_N = B_H \Rightarrow \frac{\mu_0}{4\pi}\frac{m}{r^2} = B_H$

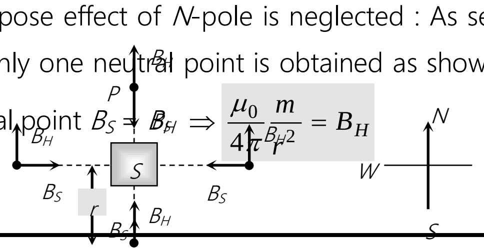


S- pole of magnet is the horizontal plane



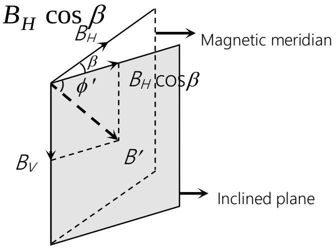
At neutral point P: $B_S - B_N \cos\theta = B_H \quad (B_S < B_N)$

If suppose effect of N-pole is neglected : As seen from top only one neutral point is obtained as shown and at neutral point $B_S = B_H \Rightarrow \frac{\mu_0}{4\pi}\frac{m}{r^2} = B_H$



Concepts

☞ *Apparent dip : In a vertical plane inclined at an angle β to the magnetic meridian, vertical component of earth's magnetic field remains uncharged while in the new inclined plane horizontal component* $B'_H = B_H \cos\beta$



$\phi' =$ *apparent angle of dip*

and $\tan\phi' = \frac{B_V}{B'_H} = \frac{B_V}{B_H \cos\beta}$

$\Rightarrow \tan\phi' = \frac{\tan\phi}{\cos\beta}$

☞ *If at any place the angle of dip is* θ and magnetic latitude is λ then tan θ = 2tanλ

☞ At the poles and equator of earth the values of total intensity are 0.66 and 0.33 *Oersted respectively.*

Examples

Example. 19 If the angles of dip at two places are 30° and 45° respectively, Then the ratio of horizontal components of earth's magnetic field at the two places will be [MP PET 1989]

(a) $\sqrt{3}:\sqrt{2}$ (b) $1:\sqrt{2}$ (c) $1:\sqrt{3}$ (d) $1:2$

Solution : (a) By using $B_H = B\cos\phi \Rightarrow \frac{(B_H)_1}{(B_H)_2} = \frac{(\cos\phi)_1}{(\cos\phi)_2} = \frac{\cos 30}{\cos 45} = \sqrt{\frac{3}{2}}$

Example. 20 At a place the earth's horizontal component of magnetic field is $0.38 \times 10^{-4}\, weber/m^2$. If the angle of dip at that place is 60°, then the vertical component of earth's field at that place in *weber/m*^{2} will be approximately

[MP PMT 1985]

(a) 0.12×10^{-4} (b) 0.24×10^{-4} (c) 0.40×10^{-4} (d) 0.62×10^{-4}

Solution : (d) By using $\tan\phi = \frac{B_V}{B_H} \Rightarrow \tan 60^o = \frac{B_V}{0.38\times10^{-4}} \Rightarrow B_V = 0.38\times10^{-4}\times\sqrt{3} = 0.62\times10^{-4}$.

Example. 21 A dip circle is so set that it moves freely in the magnetic meridian. In this position, the angle of dip is 40°. Now the dip circle is rotated so that the plane in which the needle moves makes an angle of 30° with the magnetic meridian. In this position, the needle will dip by the angle

(a) 40° (b) 30° (c) More than 40° (d) Less than 40°

Solution : (c) By using $\tan\phi' = \frac{\tan\phi}{\cos\beta}$; where $\phi = 40^o, \beta = 30^o$

As cos 30° < 1 $\Rightarrow \frac{1}{\cos 30^o} > 1$

Hence $\frac{\tan\phi'}{\tan\phi} > 1 \Rightarrow \tan\phi' > \tan\phi = \phi' > \phi$ or $\phi' > 40^o$.

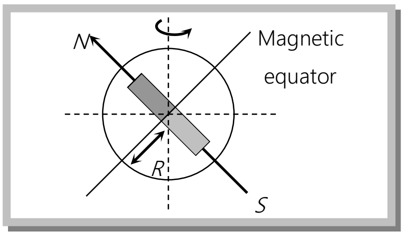
Example. 22 Earth's magnetic field may be supposed to be due to a small bar magnet located at the centre of the earth. If the magnetic field at a point on the magnetic equator is 0.3×10^{-4} *T*. Magnet moment of bar magnet is

(a) $7.8\times10^{8}\, amp \times m^2$

(b) $7.8\times10^{22}\, amp \times m^2$

(c) $6.4\times10^{22}\, amp \times m^2$

(d) None of these

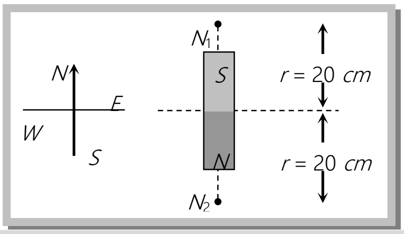


Solution : (b) When a magnet is freely suspended in earth's magnetic field, it's north pole points north, so the magnetic field of the earth may be suppose to be due to a magnetic dipole with it's south pole towards north and as equatorial point is on the broad side on position of the dipole.

$$B_e = \frac{\mu_0}{4\pi}.\frac{M}{r^3} \Rightarrow 0.3\times10^{-4} = 10^{-7}\times\frac{M}{(6.4\times\times10^{6})^3} \Rightarrow M = 7.8\times10^{22}\ A\text{-}m^2.$$

Example. 23 A short bar magnet is placed with its south pole towards geographical north. The neutral points are situated at a distance of 20 *cm* from the centre of the magnet. If $B_H = 0.3\times10^{-4}\, wb/m^2$ then the magnetic moment of the magnet is

(a) 9000 *ab- amp* × *cm*^{2}



(b) $900\ ab - amp \times cm^2$

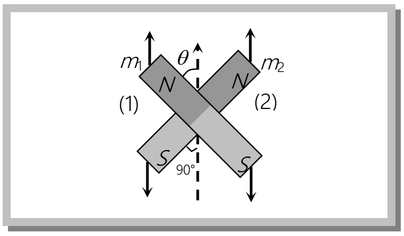
(c) $1200\ ab - amp \times cm^2$

(d) $225\ ab - amp \times cm^2$

Solution : (c) At neutral point magnetic field due to magnet = Horizontal component of earth's magnetic field

$$\Rightarrow \frac{\mu_0}{4\pi}.\frac{2M}{r^3} = B_H \Rightarrow \frac{10^{-7} \times 2 \times M \times 1}{(0.2)^3} = 0.3 \times 15^4 \Rightarrow M = 1.2 amp \times m^2 = 1200\ ab - amp \times cm^2.$$

Example: 24 Two magnets of equal mass are joined at right angles to each other as shown the magnet 1 has a magnetic moment 3 times that of magnet 2. This arrangement is pivoted so that it is free to rotate in the horizontal plane. In equilibrium what angle will the magnet 1 subtend with the magnetic meridian



(a) $\tan^{-1}\left(\frac{1}{2}\right)$

(b) $\tan^{-1}\left(\frac{1}{3}\right)$

(c) $\tan^{-1}(1)$

(d) 0°

Solution : (b) For equilibrium of the system torques on M_1 and M_2 due to B_H must counter balance each other i.e. $\vec{M}_1 \times \vec{B}_H = \vec{M}_2 \times \vec{B}_H$. If θ is the angle between M_1 and B_H then the angle between M_2 and B_H will be $(90 - \theta)$; so $M_1 B_H \sin\theta = M_2 B_H \sin(90 - \theta)$

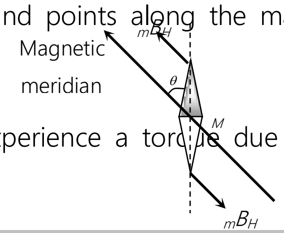
$$\Rightarrow \tan\theta = \frac{M_2}{M_1} = \frac{M}{3M} = \frac{1}{3} \Rightarrow \theta = \tan^{-1}\left(\frac{1}{3}\right)$$

Tricky Example: 2

A compass needle whose magnetic moment is 60 $amp \times m^2$ pointing geographical north at a certain place, where the horizontal component of earth's magnetic field is 40 $\mu wb/m^2$, experiences a torque $1.2 \times 10^{-3} N \times m$. What is the declination at this place

(a) 30° (b) 45° (c) 60° (d) 25°

Solution : (a) As the compass needle is free to rotate in a horizontal plane and points along the magnetic meridian,



so when it is pointing along the geographic meridian, it will experience a torque due to the horizontal component of earth's magnetic field *i.e.* $\tau = MB_H \sin\theta$

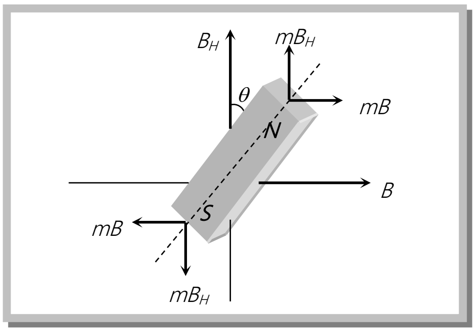
Where θ = angle between geographical

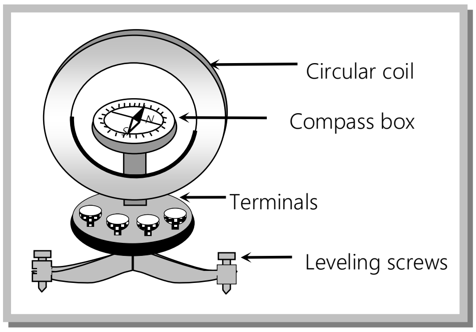
and magnetic meridians called angle of declination

So, $\sin\theta = \frac{1.2\times10^{-3}}{60\times40\times10^{-6}} = \frac{1}{2} \Rightarrow \theta = 30^o$

Tangent Law and it's Application

When a small magnet is suspended in two uniform magnetic fields B and B_H which are at right angles to each other, the magnet comes to rest at an angle θ with respect to B_H such that $B = B_H \tan\theta$. This is called tangent law.





Tangent galvanometer : It is an instrument which can detect/measure very small electric currents. It is also called as moving magnet galvanometer. It consists of three circular coils of insulated copper wire wound on a vertical circular frame made of nonmagnetic material as ebonite or wood. A small magnetic compass needle is pivoted at the centre of the vertical circular frame. This needle rotates freely in a horizontal plane inside a box made of nonmagnetic material. When the coil of the tangent galvanometer is kept in magnetic meridian and current passes through any of the coil then the needle at the centre gets deflected and comes to an equilibrium position under the action of two perpendicular field : one due to horizontal component of earth and the other due to field set up by the coil due to current (B).

In equilibrium $\mathbf{B} = \mathbf{B_H \tan\theta}$ where $B = \frac{\mu_0 ni}{2r}$; n = number of turns, r = radius of coil, i = the current to be measured, θ = angle made by needle from the direction of B_H in equilibrium.

Hence $\frac{\mu_0 Ni}{2r} = B_H \tan\theta \Rightarrow i = k\tan\theta$ where $k = \frac{2rB_H}{\mu_0 N}$ is called reduction factor.

Note : ❑ Principle of moving coil galvanometer is $i \propto \tan\theta$. Since $i \propto \tan\theta$ so it's scale is not uniform.

❑ When $\theta = 45^{o}$, reduction factor equals to current flows through coil.

❑ Sensitivity of this galvanometer is maximum at $\theta = 45^{o}$.

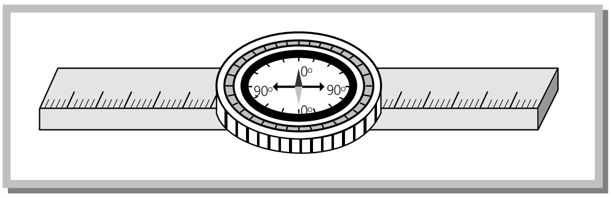
❑ This instrument is also called moving magnet type galvanometer.

Magnetic Instruments

Magnetic instruments are used to find out the magnetic moment of a bar magnet, find out the horizontal component of earth's magnetic field, compare the magnetic moments of two bar magnets.

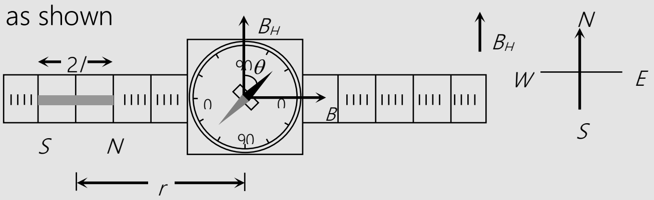
(1) **Deflection magnetometer**

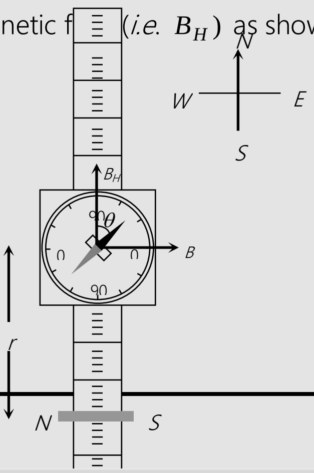
It's working is based on the principle of tangent law. It consist of a small compass needle, pivoted at the centre of a circular box. The box is kept in a wooden frame having two meter scale fitted on it's two arms. Reading of a scale at any point directly gives the distance of that point from the centre of compass needle.



Different position of deflection magnetometer : Deflection magnetometer can be used according to two following positions.

Tan A position	Tan B position
Arms of magnetometer are placed along E-W direction such that magnetic needle is acted upon by only horizontal component of earth's magnetic field (B_H) as shown	Arms of magnetometer are placed along N-S direction such that magnetic needle align itself in the direction of earth's magnetic field (*i.e.* B_H) as shown.



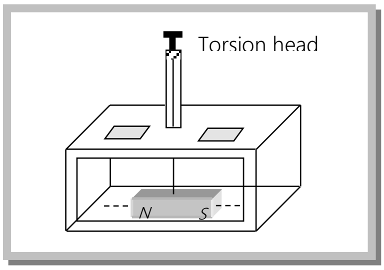


It a bar magnet is placed on one arm with it's length parallel to arm, so magnetic needle comes under the influence of two mutual perpendicular magnetic field (i) B_H and (ii) Axial magnetic field of experimental bar magnet. In equilibrium $B = B_H \tan\theta \Rightarrow \frac{\mu_0}{4\pi}\frac{2M}{r^3} = B_H \tan\theta$ (M= Magnetic moment of experimental bar magnet)	If a bar magnet is placed on one arm with it's length perpendicular to arm, so magnetic needle comes under the influence of two mutual perpendicular magnetic fields (i) B_H and (ii) equatorial magnetic field of experimental bar magnet. In equilibrium $B = B_H$ and $\Rightarrow \frac{\mu_0}{4\pi}.\frac{M}{r^3} = B_H \tan\theta$

Note : ❑ Deflection magnetometer also used to compare the magnetic moments either by deflection method or by null deflection method. **Deflection method** : $\frac{M_1}{M_2} = \frac{\tan\theta_1}{\tan\theta_2}$, **Null deflection method** : $\frac{M_1}{M_2} = \left(\frac{d_1}{d_2}\right)^3$ where d_1 and d_2 are the position of two bar magnet placed simultaneously on each arm.

(2) **Vibration magnetometer**

Vibration magnetometer is used for comparison of magnetic moments and magnetic fields. This device works on the principle, that whenever a freely suspended magnet in a uniform magnetic field, is disturbed from it's equilibrium position, it starts vibrating about the mean position.



Time period of oscillation of experimental bar magnet (magnetic moment M) in earth's magnetic field (B_H) is given by the formula. $T = 2\pi\sqrt{\frac{I}{MB_H}}$

Where, I = moment of inertia of short bar magnet $= \frac{wL^2}{12}$ (w = mass of bar magnet)

(3) **Use of vibration magnetometer**

(i) Determination of magnetic moment of a magnet :

The experimental (given) magnet is put into vibration magnetometer and it's time period T is determined.

Now $T = 2\pi\sqrt{\dfrac{I}{MB_H}} \Rightarrow M = \dfrac{4\pi^2 I}{B_H . T^2}$

(ii) Comparison of horizontal components of earth's magnetic field at two places.

$T = 2\pi\sqrt{\dfrac{I}{MB_H}}$; since I and M the magnet are constant, so $T^2 \propto \dfrac{1}{B_H} \Rightarrow \dfrac{(B_H)_1}{(B_H)_2} = \dfrac{T_2^2}{T_1^2}$

(iii) Comparison of magnetic moment of two magnets of same size and mass.

$T = 2\pi\sqrt{\dfrac{I}{M.B_H}}$; Here I and B_H are constants. So $M \propto \dfrac{1}{T^2} \Rightarrow \dfrac{M_1}{M_2} = \dfrac{T_2^2}{T_1^2}$

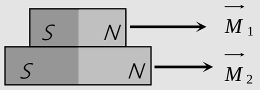
(iv) Comparison of magnetic moments of two magnets of unequal sizes and masses (by sum and difference method) :

In this method both the magnets vibrate simultaneously in two following position.

Sum position : Two magnets are placed such that their magnetic moments are additive

Net magnetic moment $M_s = M_1 + M_2$

Net moment of inertia $I_s = I_1 + I_2$



Time period of oscillation of this pair in earth's magnetic field (B_H)

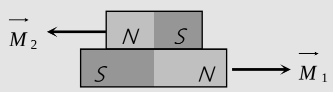
$$T_s = 2\pi\sqrt{\frac{I_s}{M_s B_H}} = 2\pi\sqrt{\frac{I_1 + I_2}{(M_1 + M_2)B_H}} \quad .....(i)$$

Frequency $\nu_s = \dfrac{1}{2\pi}\sqrt{\dfrac{M_s(B_H)}{I_s}}$

Difference position : Magnetic moments are subtractive

Net magnetic moment $M_d = M_1 + M_2$

Net moment of inertia $I_d = I_1 + I_2$



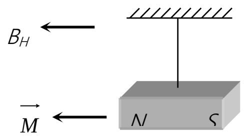
and $T_d = 2\pi\sqrt{\dfrac{I_d}{M_d B_H}} = 2\pi\sqrt{\dfrac{I_1 + I_2}{(M_1 - M_2)B_H}}$ ....(ii) and $\nu_d = \frac{1}{2\pi}\sqrt{\frac{(M_1 + M_2)B_H}{(I_1 + I_2)}}$

From equation (i) and (ii) we get $\frac{T_s}{T_d} = \sqrt{\frac{M_1 - M_2}{M_1 + M_2}} \Rightarrow \frac{M_1}{M_2} = \frac{T_d^2 + T_s^2}{T_d^2 - T_s^2} = \frac{\nu_s^2 + \nu_d^2}{\nu_s^2 - \nu_d^2}$

(v) To find the ratio of magnetic field : Suppose it is required to find the ratio $\frac{B}{B_H}$ where B is the field created by magnet and B_H is the horizontal component of earth's magnetic field.

To determine $\frac{B}{B_H}$ a primary (main) magnet is made to first oscillate in earth's magnetic field (B_H) alone and it's time period of oscillation (T) is noted.

$$T = 2\pi\sqrt{\frac{I}{M\,B_H}}$$



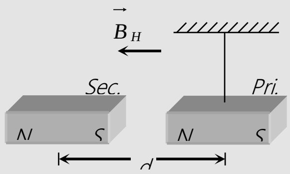
and frequency $\nu = \frac{1}{2\pi}\sqrt{\frac{M\,B_H}{I}}$

Now a secondary magnet placed near the primary magnet so primary magnet oscillate in a new field with is the resultant of B and B_H and now time period, is noted again.

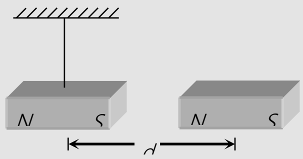
There are two important possibilities for placing secondary magnet

Possibility 1

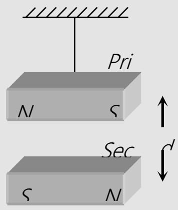
New field increases so time period of oscillation of primary magnet decreases



or



or

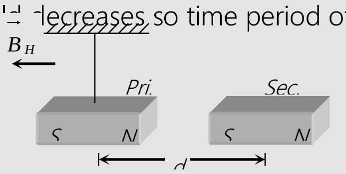


Now time period $T' = 2\pi\sqrt{\frac{I}{M(B + B_H)}}$ or new frequency $\nu' = \frac{1}{2\pi}\sqrt{\frac{M(M + B_H)}{I}}$

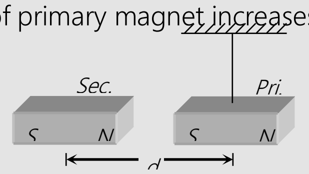
Also $\left(\frac{\nu'}{\nu}\right)^2 = \sqrt{\frac{B + B_H}{B_H}} \Rightarrow \left(\frac{\nu'}{\nu}\right)^2 = \frac{B}{B_1} + 1 \Rightarrow \frac{B}{B_H} = \left(\frac{\nu'}{\nu}\right)^2 - 1$

Possibility 2

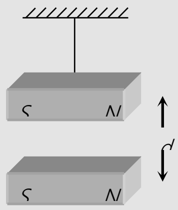
Net field decreases so time period of oscillation of primary magnet increases



or



or



$$T' = 2\pi\sqrt{\frac{I}{M(B_H - B)}} \quad (B_H > B) \quad \text{and} \quad \nu' = \frac{1}{2\pi}\sqrt{\frac{M(B_H - B)}{I}}$$

Also $\left(\frac{\nu'}{\nu}\right)^2 = \sqrt{\frac{B_H - B}{B_H}} \Rightarrow \left(\frac{\nu'}{\nu}\right)^2 = 1 - \left(\frac{B}{B_1}\right) \Rightarrow \boxed{\frac{B}{B_H} = 1 - \left(\frac{\nu'}{\nu}\right)^2}$

Concepts

- *Remember time period of oscillation in difference position is greater than that in sum position $T_d > T_s$.*
- *If a rectangular bar magnet is cut in n equal parts then time period of each part will be $\frac{1}{\sqrt{n}}$ times that of complete magnet (i.e. $T' = \frac{T}{\sqrt{n}}$) while for short magnet $T' = \frac{T}{n}$. If nothing is said then bar magnet is treated as short magnet.*
- *Suppose a magnetic needle is vibrating in earth's magnetic field. With temperature rise M decreases hence time period (T) increases but at 770°C (Curie temperature) it stops vibrating.*

Examples

Example: 25 Two magnets are held together in a vibration magnetometer and are allowed to oscillate in the earth's magnetic field. With like poles together 12 oscillations per minute are made but for unlike poles together only 4 oscillations per minute are executed. The ratio of their magnetic moments is

(a) 3 : 1 (b) 1 : 3 (c) 3 : 5 (d) 5 : 4

Solution : (d) By using $\frac{M_1}{M_2} = \frac{T_d^2 + T_s^2}{T_d^2 - T_s^2}$; where $T_s = \frac{60}{12} = 5\,sec$ and $T_d = \frac{60}{4} = 15\,sec$ $\therefore \frac{M_1}{M_2} = \frac{(15)^2 + (5)^2}{(15)^2 - (5)^2} = \frac{5}{4}$

Example: 26 The magnetic needle of a tangent galvanometer is deflected at an angle 30° due to a magnet. The horizontal component of earth's magnetic field 0.34×10^{-4} *T* is along the plane of the coil. The magnetic intensity is

[KCET 1999; AFMC 1999, 2000; BHU 2000; AIIMS 2000, 02]

(a) $1.96 \times 10^{-4} T$ (b) $1.96 \times 10^{-5} T$ (c) $1.96 \times 10^{4} T$ (d) $1.96 \times 10^{5} T$

Solution : (b) $B = B_H \tan\theta \Rightarrow B = 0.34 \times 10^{-4} \tan 30^o = 1.96 \times 10^{-5} T$

Example: 27 A magnet freely suspended in a vibration magnetometer makes 10 oscillations per minute at a place *A* and 40 oscillations per minute at a place *B*. If the horizontal component of earth's magnetic field at *A* is $36 \times 10^{-6} T$, then its value at *B* is

(a) $36 \times 10^{-6} T$ (b) $72 \times 10^{-6} T$ (c) $144 \times 10^{-6} T$ (d) $288 \times 10^{-6} T$

Solution : (c) By using $T = 2\pi\sqrt{\frac{I}{MB_H}} \Rightarrow T \propto \frac{1}{\sqrt{B_H}} \Rightarrow \frac{T_A}{T_B} = \sqrt{\frac{(B_H)_B}{(B_H)_A}} \Rightarrow \frac{60/10}{60/20} = \sqrt{\frac{(B_H)_B}{36 \times 10^{-6}}} \Rightarrow (B_H)_B = 144 \times 10^{-6} T.$

Example: 28 The magnet of a vibration magnetometer is heated so as to reduce its magnetic moment by 19%. By doing this the periodic time of the magnetometer will [MP PMT 2000]

(a) Increase by 19% (b) Increase by 11% (c) Decrease by 19% (d) Decrease by 21%

Solution : (b) $T = 2\pi\sqrt{\frac{I}{MB_H}} \Rightarrow T \propto \frac{1}{\sqrt{M}} \Rightarrow \frac{T_1}{T_2} = \sqrt{\frac{M_2}{M_1}}$

If $M_1 = 100$ then $M_2 = (100 - 19) = 81$. So, $\frac{T_1}{T_2} = \sqrt{\frac{81}{100}} = \frac{9}{10} \Rightarrow T_2 = \frac{10}{9} T_1 = 11\% T_1$

Example: 29 A magnet makes 40 oscillations per minute at a place having magnetic field intensity $B_H = 0.1 \times 10^{-5}$. At another place, it takes 2.5 sec to complete one-vibration. The value of earth's horizontal field at that place

[CPMT 1999; AIIMS 2000]

(a) $0.25 \times 10^{-6} T$ (b) $0.36 \times 10^{-6} T$ (c) $0.66 \times 10^{-8} T$ (d) $1.2 \times 10^{-6} T$

Solution : (b) By using $T = 2\pi\sqrt{\frac{I}{MB_H}} \Rightarrow \frac{T_1}{T_2} = \sqrt{\frac{(B_H)_2}{(B_H)_1}} \Rightarrow \frac{60/40}{2{,}5} = \sqrt{\frac{(B_H)_2}{0.1 \times 10^{-5}}} \Rightarrow (B_H)_2 = 0.36 \times 10^{-6} T$.

Example: 30 When 2 *amp*. current is passed through a tangent galvanometer, it gives a deflection of 30°. For 60° deflection, The current must be

(a) 1 *amp*. (b) $2\sqrt{3}$ *amp*. (c) 4 *amp*. (d) 6 *amp*.

Solution : (d) By using $i \propto \tan\theta \Rightarrow \frac{i_1}{i_2} = \frac{\tan\theta_1}{\tan\theta_2} \Rightarrow \frac{2}{i_2} = \frac{\tan 30^o}{\tan 60^o} = \frac{1}{3} \Rightarrow i_2 = 6$ *amp.*

Example. 31 In vibration magnetometer the time period of suspended bar magnet can be reduced by [CBSE PMT 1999]

(a) Moving it towards south pole (b) Moving it towards north pole

(c) Moving it toward equator (d) Any one them

Solution : (c) As we move towards equator B_H increases and it becomes maximum at equator. Hence $T = 2\pi\sqrt{\frac{I}{MB_H}}$, we can say that according to the relation *T* decreases as $B_H \uparrow$ increases (*i.e.* as we move towards equator).

Example. 32 The time period of a freely suspended magnet is 2 *sec*. If it is broken in length into two equal parts and one part is suspended in the same way, then its time period will be [MP PMT 1999]

(a) 4 *sec* (b) 2 *sec* (c) $\sqrt{2}\,sec$ (d) 1 *sec*

Solution : (d) $T = 2\pi\sqrt{\frac{I}{MB_H}}$; When a bar magnet is broken in n equal parts so magnetic moment of each part become $\frac{1}{n}$ times and moment of inertia becomes of each part becomes $\frac{1}{n^3}$ times. Hence time period becomes $\frac{1}{n}$ times *i.e.* $T' = \frac{T}{4}$

In this question $n = 2$ so, $T' = \frac{T}{2} = \frac{2}{2} = 1 sec$

Example. 33 A magnet is suspended in such a way that it oscillates in the horizontal plane. It makes 20 oscillations per minute at a place where dip angle is 30° and 15 oscillations per minute at a place where dip angle is 60°. The ratio of total earth's magnetic field at the two places is

(a) $3\sqrt{3} : 8$ (b) $16 : 9\sqrt{3}$ (c) $4 : 9$ (d) $2\sqrt{3} : 9$

Solution : (b) By using $T = 2\pi\sqrt{\frac{I}{MB_H}} = 2\pi\sqrt{\frac{I}{MB\cos\phi}}$

$$\Rightarrow T \propto \frac{1}{\sqrt{B\cos\phi}} \Rightarrow \frac{T_1}{T_2} = \sqrt{\frac{B_2}{B_1} \times \frac{\cos\phi_2}{\cos\phi_1}} \Rightarrow \frac{60/20}{60/15} = \sqrt{\frac{B_2}{B_1} \times \frac{\cos 60}{\cos 30}} \Rightarrow \frac{B_1}{B_2} = \frac{16}{9\sqrt{3}}.$$

Example. 34 If θ_1 and θ_2 are the deflections obtained by placing small magnet on the arm of a deflection magnetometer at the same distance from the compass box in tan *A* and tan *B* positions of the magnetometer respectively then the value of $\frac{\tan\theta_1}{\tan\theta_2}$ will be approximately [MP PMT 1992]

(a) 1 (b) 2 (c) $\frac{1}{2}$ (d) $\sqrt{2}$

Solution : (b) In $\tan A$ position $\frac{\mu_0}{4\pi}.\frac{2M}{d^3} = B_H \tan\theta_1$ ......(i)

In $\tan B$ position $\frac{\mu_0}{4\pi}.\frac{M}{d^3} = B_H \tan\theta_2$ .....(ii)

Dividing equation (i) by equation (ii) $\frac{\tan\theta_1}{\tan\theta_2} = \frac{2}{1}$.

Example: 35 In a vibration magnetometer, the time period of a bar magnet oscillating in horizontal component of earth's magnetic field is 2 *sec*. When a magnet is brought near and parallel to it, the time period reduces to 1 *sec*. The ratio *H/F* of the horizontal component *H* and the field *F* due to magnet will be

[MP PMT 1990]

(a) 3 (b) $\frac{1}{3}$ (c) $\sqrt{3}$ (d) $\frac{1}{\sqrt{3}}$

Solution : (b) Time period decreases *i.e.* field due to magnet (*F*) assist the horizontal component of earth's magnetic field (see theory)

Hence by using $\frac{B}{B_H} = \left(\frac{T}{T'}\right) - 1 \Rightarrow \frac{F}{H} = \left(\frac{2}{1}\right)^2 - 1 = 3 \Rightarrow \frac{H}{F} = \frac{1}{3}$.

Example: 36 A certain amount of current when flowing in a properly set tangent galvanometer, produces a deflection of 45°. If the current be reduced by a factor of $\sqrt{3}$, the deflection would

(a) Decrease by 30° (b) Decreases by 15° (c) Increase by 15° (d) Increase by 30°

Solution : (b) By using $i \propto \tan\theta \Rightarrow \frac{i_1}{i_2} = \frac{\tan\theta_1}{\tan\theta_2} \Rightarrow \frac{i_1}{i_1/\sqrt{3}} = \frac{\tan 45^o}{\tan\theta_2} \Rightarrow \sqrt{3}\tan\theta_2 = 1 \Rightarrow \tan\theta_2 = \frac{1}{\sqrt{3}} \Rightarrow \theta_2 = 30^o$

So deflection will decrease by $45^o - 30^o = 15^o$.

Example: 37 The angle of dip at a place is 60°. A magnetic needle oscillates in a horizontal plane at this place with period *T*. The same needle will oscillate in a vertical plane coinciding with the magnetic meridian with a period

(a) *T* (b) 2*T* (c) $\frac{T}{2}$ (d) $\frac{T}{\sqrt{2}}$

Solution : (d) When needle oscillates in horizontal plane

Then it's time period is $T = 2\pi\sqrt{\frac{I}{MB_H}}$ ......(i)

When needle oscillates in vertical plane *i.e.* It oscillates in total earth's total magnetic field (B)

Hence $T' = 2\pi\sqrt{\frac{I}{M}}$ ......(ii)

Dividing equation (ii) by (i) $\frac{T'}{T} = \sqrt{\frac{B_H}{B}} = \sqrt{\frac{B\cos\phi}{B}} = \sqrt{\cos 60} = \frac{1}{\sqrt{2}} \Rightarrow T' = \frac{T}{\sqrt{2}}$

Example. 38 A dip needle vibrates in the vertical plane perpendicular to the magnetic meridian. The time period of vibration is found to be 2 *seconds*. The same needle is then allowed to vibrate in the horizontal plane and the time period is again found to be 2 *seconds*. Then the angle of dip is

(a) 0° (b) 30° (c) 45° (d) 90°

Solution : (c) In vertical plane perpendicular to magnetic meridian.

$T = 2\pi\sqrt{\frac{I}{MB_V}}$ ......(i)

In horizontal plane $T = 2\pi\sqrt{\frac{I}{MB_H}}$ .....(ii)

Equation (i) and (ii) gives $B_V = B_H$

Hence by using $\tan\phi = \frac{B_V}{B_H} \Rightarrow \tan\phi = 1 \Rightarrow \phi = 45^o$

Tricky Example: 3

A magnet is suspended horizontally in the earth's magnetic field. When it is displaced and then released it oscillates in a horizontal plane with a period *T*. If a place of wood of the same moment of inertia (about the axis of rotation) as the magnet is attached to the magnet what would the new period of oscillation of the system become

(a) $\frac{T}{3}$ (b) $\frac{T}{2}$ (c) $\frac{T}{\sqrt{2}}$ (d) $T\sqrt{2}$

Solution : (d) Due to wood moment of inertia of the system becomes twice but there is no change magnetic moment of the system.

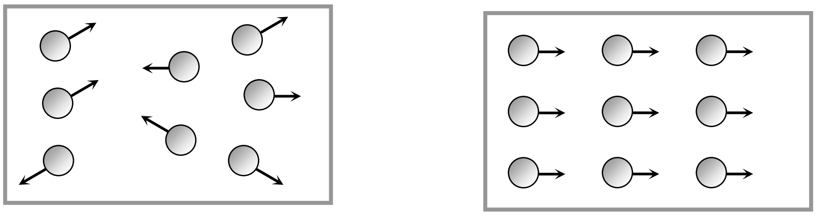
Hence by using $T = 2\pi\sqrt{\frac{I}{MB_H}} \Rightarrow T \propto \sqrt{I} \Rightarrow T' = \sqrt{2}\,T$

Magnetic Materials

(1) **Types of magnetic material :** On the basis of mutual interactions or behaviour of various materials in an external magnetic field, the materials are divided in three main categories.

(i) Diamagnetic materials : Diamagnetism is the intrinsic property of every material and it is generated due to mutual interaction between the applied magnetic field and orbital motion of electrons.

(ii) Paramagnetic materials : In these substances the inner orbits of atoms are incomplete. The electron spins are uncoupled, consequently on applying a magnetic field the magnetic moment generated due to spin motion align in the direction of magnetic field and induces magnetic moment in its direction due to which the material gets feebly magnetised. In these materials the electron number is odd.



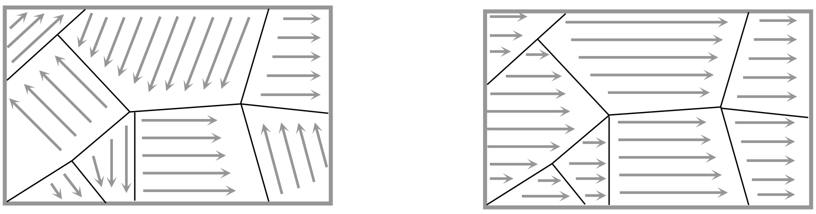
(a)
When no field is applied

(b)
On application of field (B)

(iii) Ferromagnetic materials : In some materials, the permanent atomic magnetic moments have strong tendency to align themselves even without any external field.

These materials are called ferromagnetic materials.

In every unmagnetised ferromagnetic material, the atoms form domains inside the material. The atoms in any domain have magnetic moments in the same direction giving a net large magnetic moment to the domain. Different domains, however, have different directions of magnetic moment and hence the materials remain unmagnetised. On applying an external magnetic field, these domains rotate and align in the direction of magnetic field.



Unmagnetised

Magnetised

(2) **Curie Law** : The magnetic susceptibility of paramagnetic substances in inversely to its absolute temperature *i.e.* $\chi \propto \frac{1}{T} \Rightarrow \chi \propto \frac{C}{T}$

where C = Curie constant, T = absolute temperature

On increasing temperature, the magnetic susceptibility of paramagnetic materials decreases and vice versa.

The magnetic susceptibility of ferromagnetic substances does not change according to Curie law.

(i) Curie temperature (T_c) : The temperature above which a ferromagnetic material behaves like a paramagnetic material is defined as Curie temperature (T_c).

or

The minimum temperature at which a ferromagnetic substance is converted into paramagnetic substance is defined as Curie temperature.

For various ferromagnetic materials its values are different, *e.g.* for *Ni*, $T_{C_{Ni}} = 358\,^oC$

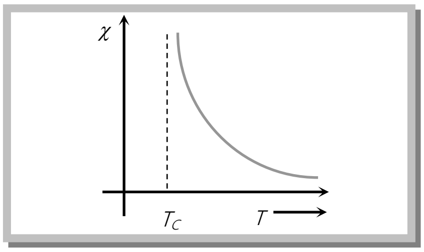
for *Fe*, $T_{C_{Fe}} = 770\,^oC$

for *CO*, $T_{C_{CO}} = 1120\,^oC$

At this temperature the ferromagnetism of the substances suddenly vanishes.

(ii) Curie-weiss law : At temperatures above Curie temperature the magnetic susceptibility of ferromagnetic materials is inversely proportional to ($T - T_c$) *i.e.* $\chi \propto \frac{1}{T - T_c}$

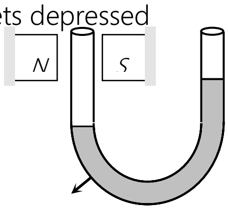
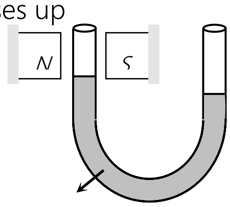
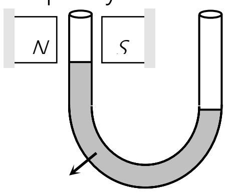
$\Rightarrow \chi = \frac{C}{(T - T_c)}$ Here T_c = Curie temperature



χ-T curve is shown (for Curie-Weiss Law)

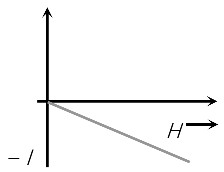
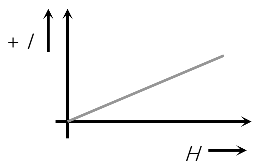
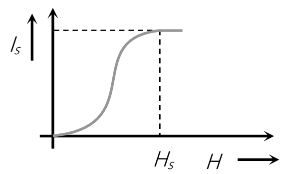
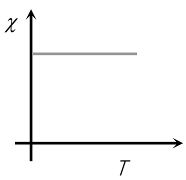
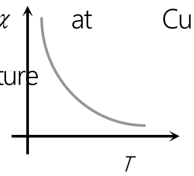
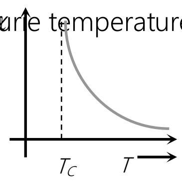
(3) **Comparative study of magnetic materials**

Property	Diamagnetic substances	Paramagnetic substances	Ferromagnetic substances
Cause of magnetism	Orbital motion of electrons	Spin motion of electrons	Formation of domains

Explanation of magnetism	On the basis of orbital motion of electrons	On the basis of spin and orbital motion of electrons	On the basis of domains formed
Behaviour In a non-uniform magnetic field	These are repelled in an external magnetic field *i.e.* have a tendency to move from high to low field region.	These are feebly attracted in an external magnetic field *i.e.*, have a tendency to move from low to high field region	These are strongly attracted in an external magnetic field *i.e.* they easily move from low to high field region
State of magnetisation	These are weekly magnetised in a direction opposite to that of applied magnetic field	These get weekly magnetised in the direction of applied magnetic field	These get strongly magnetised in the direction of applied magnetic field
When the material in the form of liquid is filled in the *U*-tube and placed between pole pieces.	Liquid level in that limb gets depressed 	Liquid level in that limb rises up 	Liquid level in that limb rises up very much 
On placing the gaseous materials between pole pieces	The gas expands at right angles to the magnetic field.	The gas expands in the direction of magnetic field.	The gas rapidly expands in the direction of magnetic field
The value of magnetic induction B	$B < B_0$	$B > B_0$	$B >> B_0$

where B_0 is the magnetic induction in vacuum

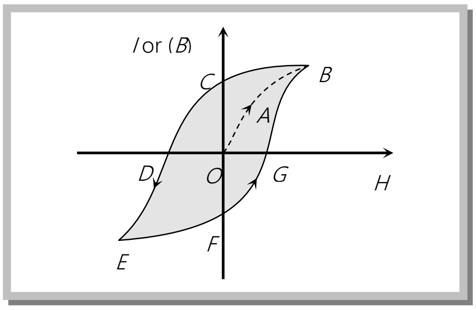
Magnetic susceptibility χ	Low and negative $\lvert\chi\rvert \approx 1$	Low but positive $\chi \approx 1$	Positive and high $\chi \approx 10^2$
Dependence of χ on temperature	Does not depend on temperature (except *Bi* at low temperature)	Inversely proportional to temperature $\chi \propto \frac{1}{T}$ or	$\chi \propto \frac{1}{T - T_c}$ or $\chi = \frac{C}{T - T_c}$ This is called

		$\chi = \frac{C}{T}$. This is called Curie law, where C = Curie constant	Curie Weiss law. T_c = Curie temperature
Dependence of χ on H	Does not depend independent	Does not depend independent	Does not depend independent
Relative permeability (μ_r)	$\mu_r < 1$	$\mu_r > 1$	$\mu_r >> 1$ $\mu_r = 10^2$
Intensity of magnetisation (I)	I is in a direction opposite to that of H and its value is very low	I is in the direction of H but value is low	I is in the direction of H and value is very high.
I-H curves			
Magnetic moment (M)	The value of M is very low (≈ 0 and is in a direction opposite to H.)	The value of M is very low and is in the direction of H	The value of M is very high and is in the direction of H
Transition of materials (at Curie temperature)	These do not change. 	On cooling, these get converted to ferromagnetic materials at Curie temperature 	These get converted into paramagnetic materials above Curie temperature 
The property of magnetism	Diamagnetism is found in those materials the atoms of which have even	Paramagnetism is found in those materials the atoms of which have majority of	Ferro-magnetism is found in those materials which when placed in an external

	number electrons	electron spins in the same direction	magnetic field are strongly magnetised
Examples	*Cu, Ag, Au, Zn, Bi, Sb, NaCl,* H_2O air and diamond *etc.*	*Al, Mn, Pt, Na,* $CuCl_2$, O_2 and crown glass	*Fe, Co, Ni, Cd,* Fe_3O_4 *etc.*
Nature of effect	Distortion effect	Orientation effect	Hysteresis effect

(4) **Hysteresis :** For ferromagnetic materials, by removing external magnetic field *i.e.* $H = 0$. The magnetic moment of some domains remain aligned in the applied direction of previous magnetising field which results into a residual magnetism.

The lack of retracibility as shown in figure is called hysteresis and the curve is known as hysteresis loop.



(i) When magnetising field (H) is increased from O, the intensity of magnetisation I increases and becomes maximum. This maximum value is called the saturation value.

(ii) When H is reduced, I reduces but is not zero when $H = 0$. The remainder value OC of magnetisation when $H = 0$ is called the residual magnetism or retentivity.

The property by virtue of which the magnetism (I) remains in a material even on the removal of magnetising field is called Retentivity or Residual magnetism.

(iii) When magnetic field H is reversed, the magnetisation decreases and for a particular value of H, denoted by H_c, it becomes zero *i.e.*, $H_c = OD$ when $I = 0$. This value of H is called the corecivity.

(iv) So, the process of demagnetising a material completely by applying magnetising field in a negative direction is defined Corecivity. Corecivity assesses the softness or hardness of a magnetic material. Corecivity signifies magnetic hardness or softness of substance :

Magnetic hard substance (steel) → High corecvity

Magnetic soft substance (soft iron) → Low corecivity

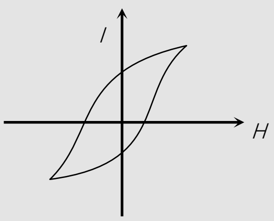
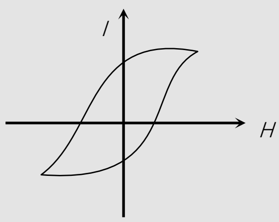
(v) When field H is further increased in reverse direction, the intensity of magnetisation attains saturation value in reverse direction (*i.e.* point E)

(vi) When H is decreased to zero and changed direction in steps, we get the part $EFGB$.

Thus complete cycle of magnetisation and demagnetisation is represented by $BCDEFGB$.

Note : ❑ The energy loss (or hysteresis energy loss) in magnetising and demagnetising a specimen is proportional to the area of hysteresis loop.

(vii) Comparison between soft iron and steel :

Soft iron	Steel
	
The area of hysteresis loop is less (low energy loss)	The area of hysteresis loop is large (high energy loss)

Less relativity and corecive force	More retentivity and corecive force
Magnetic permeability is high	Magnetic permeability is less
Magnetic susceptibility (χ) is high	χ is low
Intensity of magnetisation (I) is high	I is low
It magnetised and demagnetised easily	Magnetisation and demagnetisation is complicated
Used in dynamo, transformer, electromagnet tape recorder and tapes *etc.*	Used for making permanent magnet.

Concepts

- *An iron cored coil and a bulb are connected in series with an ac generator. If an iron rod is introduced inside a coil, then the intensity of bulb will decrease, because some energy lest in magnetising the rod.*
- *Hysteresis energy loss = Area bound by the hysteresis loop = VAnt Joule*

  Where , V = Volume of ferromagnetic sample, A = Area of B – H loop P, n = Frequency of alternating magnetic field and t = Time.

Examples

Example: 39 A ferromagnetic substance of volume 10^{-3} m^3 is placed in an alternating field of 50 *Hz*. Area of hysteresis curve obtained is 0.1 M.K.S. unit. The heat produced due to energy loss per second in the substance will be

(a) 5 *J* (b) 5×10^{-2} *cal* (c) 1.19×10^{-3} *cal* (d) No loss of energy

Solution : (c) By using heat loss = *VAnt*; whre *V* = volume = 10^{-3} m^3; *A* = Area = 0.1m^2, *n* = frequency = 50 *Hz* and *t* = time = 1*sec*

Heat loss = $10^{-3} \times 0.1 \times 50 \times 1 = 5 \times 10^{-3}$ *J* = 1.19×10^{-3} *cal*

Example: 40 A magnetising field of 1600 *A*-m^{-1} produces a magnetic flux of 2.4×10^{-5} *Wb* in an iron bar of cross-sectional area 0.2 cm^2. The susceptibility of an iron bar is

(a) 298 (b) 596 (c) 1192 (d) 1788

Solution : (b) By using $B = \mu H = \mu_0 \mu_r H$ and $\mu_r = (1 + \chi_m) \Rightarrow \mu_r = \frac{B}{\mu_0 H} = \frac{\phi}{\mu_0 HA}$

$$\mu_r = \frac{2.4\times10^{-5}}{(4\pi\times10^{-7})\times1600\times(0.2\times10^{-4})} = 596.8.\ \text{Hence}\ \chi_m = 595.8 \approx 596$$

Example. 41 For iron it's density is 7500 kg/m^3 and mass 0.075 *kg*. If it's magnetic moment is 8×10^{-7} $Amp \times m^2$, it's intensity of magnetisation is

(a) 8 *Amp/m* (b) 0.8 *Amp/m* (c) 0.08 *Amp/m* (d) 0.008 *Amp/m*

Solution : (c) $I = \frac{M}{V} = \frac{Md}{m} = \frac{8\times10^{-7}\times7500}{0.075} = 0.08\ Amp/m$

Example. 42 The dipole moment of each molecule of a paramagnetic gas is 1.5×10^{-23} $Amp \times m^2$. The temperature of gas is 27°*C* and the number of molecules per unit volume in it is 2×10^{26} m^{-3}. The maximum possible intensity of magnetisation in the gas will be

(a) 3×10^3 *Amp/m* (b) 4×10^{-3} *Amp/m* (c) 5×10^5 *Amp/m* (d) 6×10^{-4} *Amp/m*

Solution : (a) $I = \frac{M}{V} = \frac{\mu N}{V} = \frac{1.5\times10^{-23}\times2\times10^{26}}{1} = 3\times10^3\ Amp/m$

Example. 43 The coereivity of a small bar magnet is 4×10^3 *Amp/m*. It is inserted inside a solenoid of 500 *turns* and length 1 *m* to demagnetise it. The amount of current to be passed through the solenoid will be

(a) 2.5 *A* (b) 5 *A* (c) 8 *A* (d) 10 *A*

Solution : (c) $H = ni \Rightarrow i = \frac{H}{n} = \frac{4\times10^3}{500} = 8A$

Example. 44 The units for molar susceptibility

(a) m^3 (b) $kg\text{-}m^{-3}$ (c) $kg^{-1}\ m^3$ (d) No units

Solution : (a) Molar susceptibility $= \frac{\text{Volume susceptibility}}{\text{Density of material}} \times \text{molecular weight} = \frac{I/H}{\rho}\times M = \frac{I/H}{M/V}\times M$

So it's unit is m^3.

Example. 45 The ratio of the area of *B*-*H* curve and *I*-*H* curve of a substance in M.K.S. system is

(a) μ_0^2 (b) $\frac{1}{\mu_0^2}$ (c) μ_0 (d) $\frac{1}{\mu_0}$

Solution : (c) Area of *B*-*H* loop $= \mu_0$ (Area of *I*-*H*)